1)

$$\frac{k}{k-1}\frac{p_0}{\rho_0} = \frac{k}{k-1}\frac{p}{\rho} + \frac{V^2}{2}$$

$$\frac{p_0}{\rho_0} = \frac{p}{\rho} + \frac{k-1}{k}\frac{V^2}{2}$$

$$\frac{p_0}{\rho_0}\frac{\rho}{p} = 1 + \frac{k-1}{k}\frac{\rho}{p}\frac{V^2}{2}$$

isentropic : $\frac{p}{\rho^k} = \frac{p_0}{\rho^k}$

$$\rho_0 = \left(\frac{p_0}{p}\right)^{1/k}\rho$$

$$\frac{p_0}{\cancel{\rho}}\left(\frac{p}{p_0}\right)^{1/k}\frac{\cancel{\rho}}{p} = 1 + \frac{k-1}{k}\frac{\rho}{p}\frac{V^2}{2}$$

$$\left(\frac{p_0}{p}\right)^{\frac{k-1}{k}} = 1 + \frac{k-1}{k}\frac{\rho}{p}\frac{V^2}{2}$$

$$\boxed{p_0 = p\left[1 + \frac{k-1}{k}\frac{\rho}{p}\frac{V^2}{2}\right]^{k/(k-1)}}$$

Assy #5 #2

$$\vec{V}_1 = \frac{Q}{A_1} = \frac{0.1\,m^3/s}{\frac{\pi}{4}(0.15m)^2}\,\hat{\imath} = 5.66\,m/s\,\hat{\imath}$$

$$\vec{V}_2 = \frac{Q}{A_2}(-\hat{\jmath}) = \frac{0.1\,m^3/s}{\pi/4(0.1m)^2}(-\hat{\jmath}) = -12.73\,m/s\,\hat{\jmath}$$

$$(P_2 = P_a)$$

Bernoulli: $$\frac{P_1}{\rho} + gz_1 + \frac{|V_1|^2}{2} = \frac{P_a}{\rho} + gz_2 + \frac{|V_2|^2}{2} \qquad z_1 \approx z_2$$

$$\frac{P_1 - P_a}{\rho} = \frac{1}{2}\left(|V_1|^2 - |V_2|^2\right) \qquad P_1 - P_a = \underbrace{(500\,kg/m^3)}_{\rho/2}\left[(12.73m/s)^2 - (5.66m/s)^2\right]$$

$$P_1 - P_a = 65\,kPa$$

$$\sum F = \cancelto{0}{\frac{\partial}{\partial t}\iiint_V \rho\vec{v}\,dV} + \iint_S \rho\vec{v}\,\vec{v}\cdot\hat{n}\,ds$$

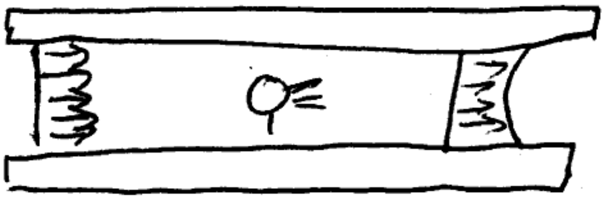
$$F_R + F_P = \iint_S \rho\vec{v}\,\vec{v}\cdot\hat{n}\,ds$$

$$F_R = -\rho\vec{V}_1 Q + \rho\vec{V}_2 Q - (P_1 - P_a)S_1\,\hat{\imath}$$

$$= (1000\,kg/m^3)(0.1\,m^3/s)\left[-(5.66m/s\,\hat{\imath}) + (-12.73m/s)\hat{\jmath}\right] - 65000\,Pa\,\frac{\pi}{4}(0.15m)^2\,\hat{\imath}$$

$$= \left[-1715\,\hat{\imath} - 1273\,\hat{\jmath}\right]N$$

Assgn #4 #2



$\rho = 1.225\ kg/m^3$

$R = 0.5m$

$V_A = 20 m/s$

$V_B = a + 2a\left(\frac{r}{R}\right)^2$

$P_A - P_B = 100 Pa$

conservation of mass

$$0 = \underbrace{\cancel{\frac{\partial}{\partial t}\iiint_V \rho\, dV}}_{\to 0} + \iint_S \rho \vec{v}\cdot n\, dS$$

$$0 = -\rho V_A S + \iint_{S_B} \rho\left[a + 2a\left(\frac{r}{R}\right)^2\right] dS$$

$$\rho V_A S = \int_0^{2\pi}\int_0^R \rho\left[a + 2a\left(\frac{r}{R}\right)^2\right] r\, dr\, d\theta$$

$$\rho V_A \pi R^2 = 2\pi\rho a \int_0^R \left[1 + 2\left(\frac{r}{R}\right)^2\right] r\, dr$$

$$= 2\pi\rho a \int_0^R \left[r + 2\frac{r^3}{R^2}\right] dr$$

$$= 2\pi\rho a \left[\frac{r^2}{2} + \frac{r^4}{2R^2}\right]_0^R$$

$$= 2\pi\rho a \left[R^2\right]$$

$$\cancel{\rho} V_A \cancel{\pi} \cancel{R^2} = 2\cancel{\pi}\cancel{\rho} a \cancel{R^2}$$

$$\boxed{a = \frac{V_A}{2}}$$

$$\sum F_x = \frac{\partial}{\partial t} \cancel{\iiint_V \rho V_x \, dV'} + \iint_S \rho V_x \vec{V} \cdot n \, dS$$

$$= -\rho V_A^2 \pi R^2 + \int_0^{2\pi} \int_0^R \rho \left(\frac{V_A}{2}\right)^2 \left[1 + 2\left(\frac{r}{R}\right)^2\right]^2 r \, dr \, d\theta$$

$$= -\rho V_A^2 \pi R^2 + \frac{\rho V_A^2}{4} (2\pi) \int_0^R \left[1 + 2\left(\frac{r}{R}\right)^2\right]^2 r \, dr$$

$$= -\rho V_A^2 \pi R^2 + \frac{\rho V_A^2}{2} \pi \int_0^R \left[1 + 4\frac{r^2}{R^2} + 4\frac{r^4}{R^4}\right] r \, dr$$

$$= -\rho V_A^2 \pi R^2 + \frac{\rho V_A^2}{2} \pi \int_0^R \left[r + 4\frac{r^3}{R^2} + 4\frac{r^5}{R^4}\right] dr$$

$$= -\rho V_A^2 \pi R^2 + \frac{\rho V_A^2}{2} \pi \left[\frac{r^2}{2} + 4\frac{r^4}{4R^2} + \frac{4}{6}\frac{r^6}{R^4}\right]_0^R$$

$$= -\rho V_A^2 \pi R^2 + \frac{\rho V_A^2}{2} \pi \left[\frac{R^2}{2} + R^2 + \frac{2}{3}R^2\right]$$

$$= -\rho V_A^2 \pi R^2 + \frac{\rho V_A^2}{2} \pi \left[\frac{13}{6} R^2\right]$$

$$= \rho V_A^2 \pi R^2 \text{ [redacted] } \frac{1}{12}$$

$$F_R + F_p = \rho V_A^2 \pi R^2 \text{ [redacted] } \frac{1}{12}$$

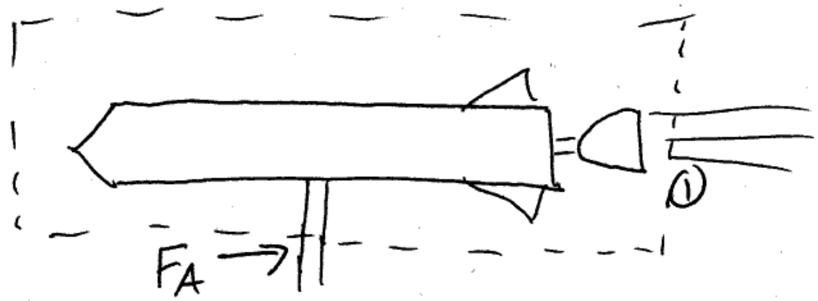
$$F_R + [P_A - P_B] \pi R^2 = \rho V_A^2 \pi R^2 \text{ [redacted] } \frac{1}{12}$$

$$F_R = \left[\rho V_A^2 \text{ [redacted] } \left(\frac{1}{12}\right) - (P_A - P_B)\right] \pi R^2$$

$$\boxed{-46\,N\uparrow}$$

$$F_R = \left[(1.225\,kg/m^3)(20\,m/s)^2 \left(\frac{1}{12}\right) - (100\,Pa)\right] \pi (0.5\,m)^2 = \text{[redacted]}$$

3)



$$0 = \frac{\partial}{\partial t} \iiint_{C.V.} \rho \, dV + \oiint_S \rho \text{[redacted]} \vec{V} \cdot \hat{n} \, dS$$

$$\frac{\partial}{\partial t} \iiint_{C.V.} \rho \, dV = -0.5 \text{ kg/s} \quad \boxed{\text{stated in problem}}$$

$$0 = -0.5 \text{ kg/s} + \oiint_S \rho \vec{V} \cdot \hat{n} \, dS$$

$$0.5 \text{ kg/s} = \rho_{exit} V_{exit} S_{exit}$$

$$\rho_{exit} = \frac{0.5 \text{ kg/s}}{(1970 \text{ m/s})(7 \times 10^{-4} \text{ m}^2)} = 0.363 \text{ kg/m}^3$$

Assume momentum in contol volume constant

$$\sum \vec{F} = \cancel{\frac{\partial}{\partial t} \iiint_{C.V.} \rho \vec{V} \, dV} + \oiint_S \rho \vec{V} \, \vec{V} \cdot \hat{n} \, dS$$

x - dir :

$$F_A = \iint_{S_1} \rho \vec{U}\, \vec{V} \cdot \hat{n}\, dS$$

$$= \rho_{exit} V_{exit}^2 S_{exit}$$

$$= (0.363\ kg/m^3)(1970\ m/s)^2(7\times10^{-4}\ m^2)$$

$$\boxed{= 985\ N}$$